Exercise 22

If $g(x) + x \sin g(x) = x^2$, find $g'(0)$.

Solution

Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}[g(x) + x \sin g(x)] = \frac{d}{dx}(x^2)$$

$$\frac{d}{dx}[g(x)] + \frac{d}{dx}[x \sin g(x)] = 2x$$

$$g'(x) + \left[\frac{d}{dx}(x)\right] \sin g(x) + x \left[\frac{d}{dx} \sin g(x)\right] = 2x$$

$$g'(x) + (1) \sin g(x) + x \left\{[\cos g(x)] \cdot \frac{d}{dx}[g(x)]\right\} = 2x$$

$$g'(x) + \sin g(x) + x g'(x) \cos g(x) = 2x$$

Solve for $g'(x)$.

$$g'(x)[1 + x \cos g(x)] = 2x - \sin g(x)$$

$$g'(x) = \frac{2x - \sin g(x)}{1 + x \cos g(x)}$$

Evaluate it at $x = 0$.

$$g'(0) = \frac{2(0) - \sin g(0)}{1 + (0) \cos g(0)}$$

$$= -\sin g(0)$$